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Control of the PTO system of OWCs: feedback vs model predictive control

João C. C. Henriques and A. F. O. Falcão

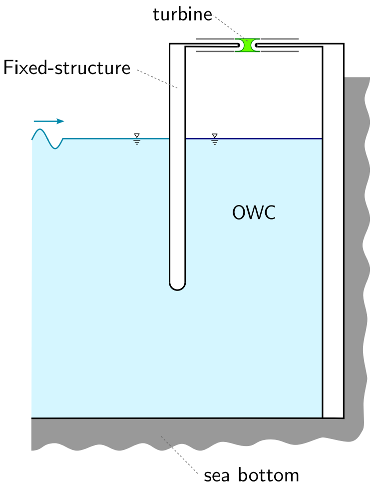
joaochenriques@tecnico.ulisboa.pt
antonio.falcao@tecnico.ulisboa.pt

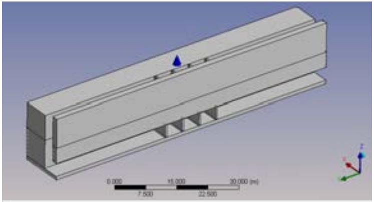
Outline of the presentation

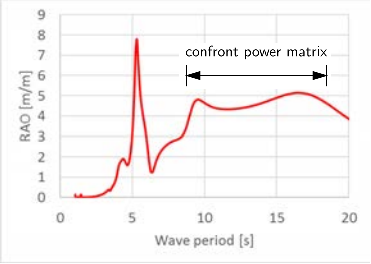
- **PART 1**
  - Generator feedback control: **Mutriku** test case
    - Power take-off system: biradial vs Wells turbine
    - Mathematical model
    - Generator feedback control law: computation and sensitivity analysis
- **PART 2**
  - Model predictive latching control: **Spar-buoy OWC** test case
  - Power take-off system: biradial turbine equipped with an **HSSV**
  - Changes to mathematical model
  - A discrete control algorithm based on Pontryagin's Maximum Principle
  - A **new** continuous control method based on the Discontinuous Galerkin Method
- Major conclusions

PART1 - Mutriku power plant test case

Hydrodynamic model

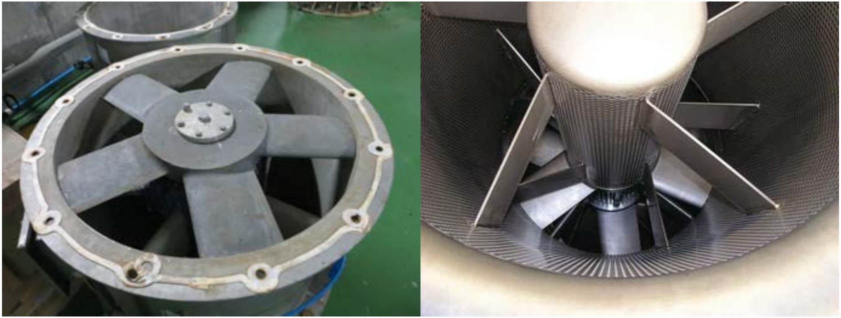






- Hydrodynamic model developed by Wanan Sheng
- High-order method, 2192 variables, 300 frequencies

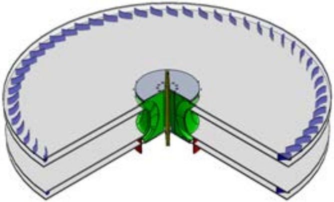
- Wells turbine with biplane rotor without guide vanes

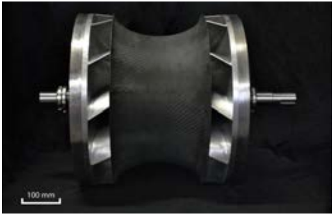


- Rotor diameter: 0.75 m. Generator rated power: 18.5 kW

Biradial turbine to be installed at Mutriku within OPERA H2020 Project

- Biradial turbine with fixed guide vanes (Kymaner/IST patent WO/2011/102746)





- Rotor diameter: 0.50 m. Generator rated power: 30.0 kW
- To be installed at Mutriku in mid-April 2017 and in the Oceantec spar-buoy, deployed at BIMEP site, in Sept. 2017 (OPERA H2020 Project)

Turbine dimensionless curves

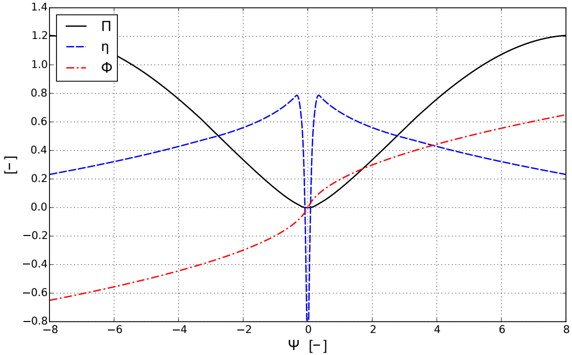
- The performance characteristics of a turbine in dimensionless form

$$\Psi = \frac{p}{\rho_{in}\,\Omega^2\, d^2}$$

$$\Phi = \frac{\dot{m}_{turb}}{\rho_{in}\,\Omega\, d^3}$$

$$\Pi = \frac{P_{turb}}{\rho_{in}\,\Omega^3\, d^5}$$

$$\eta = \frac{\Pi}{\Psi\Phi}$$



- d - rotor diameter

- For Re $> 10^6$ and Ma < 0.3 the dimensionless curves Φ, Π and η are independent of Re and Ma

$$\Rightarrow \quad \dot{m}_{turb}\,(p, \Omega, \rho_{in}) = \rho_{in}\,\Omega\, d^3\, \Phi(\Psi), \qquad P_{turb}\,(p, \Omega, \rho_{in}) = \rho_{in}\,\Omega^3\, d^5\, \Pi(\Psi)$$

- **OWC motion is modelled as a rigid piston** $(L_{\mathrm{OWC}} \ll \lambda_{\mathrm{waves}})$

$$\left(m_2 + A_{22}^{\infty}\right) \ddot{x}_2 = \underbrace{-\rho_w g S_2 \, x_2}_{\text{buoyancy}} \underbrace{-R_{22}}_{\text{radiation}} \underbrace{+F_{d2}}_{\text{diffraction}} \underbrace{-p_{\mathrm{atm}} \, S_2 \, p^*}_{\text{air chamber}}$$

- **Air chamber pressure** (see [1, 2])

$$\dot{p}^* = -\gamma \left(p^* + 1\right) \frac{\dot{V}_c}{V_c} - \gamma \left(p^* + 1\right)^{(\gamma-1)/\gamma} \frac{\dot{m}_{\mathrm{turb}}}{\rho \, V_c}$$

- **Turbine/generator set dynamics**

$$I \dot{\Omega}^* = \left(T_{\mathrm{turb}} - T_{\mathrm{gen}}\right) \Omega_{\max}^{-1} = \left(\rho_{\mathrm{in}} \, \Omega^2 d^5 \, \Pi(\Psi) - T_{\mathrm{gen}} \right) \Omega_{\max}^{-1}$$

- Dimensionless variables
  - $p^* = \dfrac{p - p_{\mathrm{atm}}}{p_{\mathrm{atm}}}$ and $\Omega^* = \Omega / \Omega_{\max}$
  - $\mathcal{O}\left[x_2\right] \approx \mathcal{O}\left[p^*\right] \approx \mathcal{O}\left[\Omega^*\right] \approx 1 \quad \Rightarrow$ Similar orders of magnitude for errors

- **Generator feedback control law**
  - If the turbine operates at the best efficiency point $\eta_{\max}$

$$P_{\text{turb}} \approx \underbrace{\rho_{\text{atm}}\, d^5\, \Pi(\,\Psi(\eta_{\max})\,)}_{\text{const}}\ \Omega^3 = \text{const}\,\Omega^3$$

the generator should follow the turbine in time-average

Feedback control law

$$P_{\text{gen}} = \min\left(\, a\,\Omega^b,\ P_{\text{gen}}^{\text{rated}}\,\right)$$

$$T_{\text{gen}} = \min\left(\, a\,\Omega^{b-1},\ \frac{P_{\text{gen}}^{\text{rated}}}{\Omega}\,\right)$$

  - The control law is clipped at generator rated power

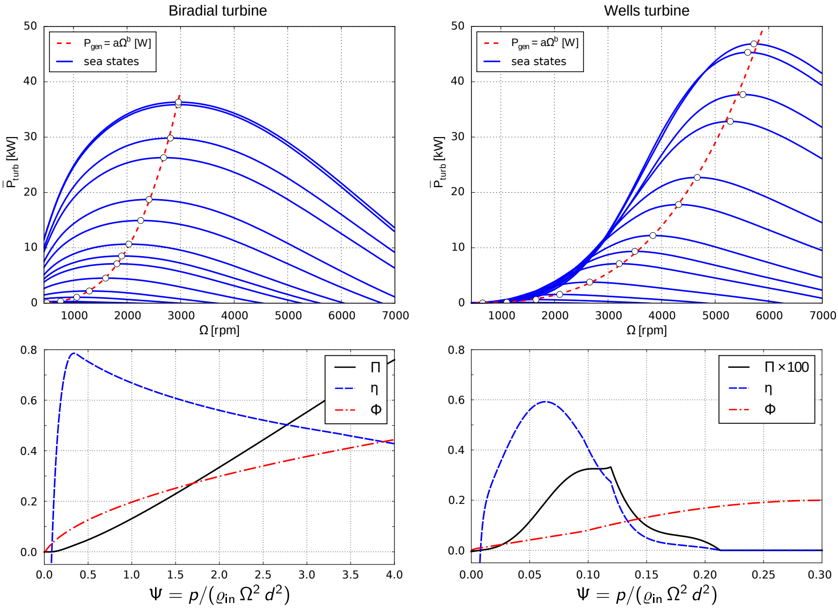
Generator feedback control law

- Constants "a" and "b" are functions of the WEC hydrodynamics and turbine geometry **but not of sea-state**
- Usually $2.4 < b < 3.6$
- To compute "a" and "b" we use a constant rotational speed model

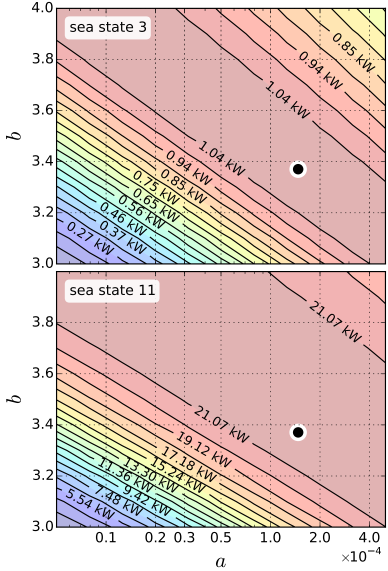
  $\dot{\Omega}^* = 0 \quad \Rightarrow \quad$ very large inertia

  for the Mutriku wave climate (14 sea states)
- Further details can be found in [1, 2]

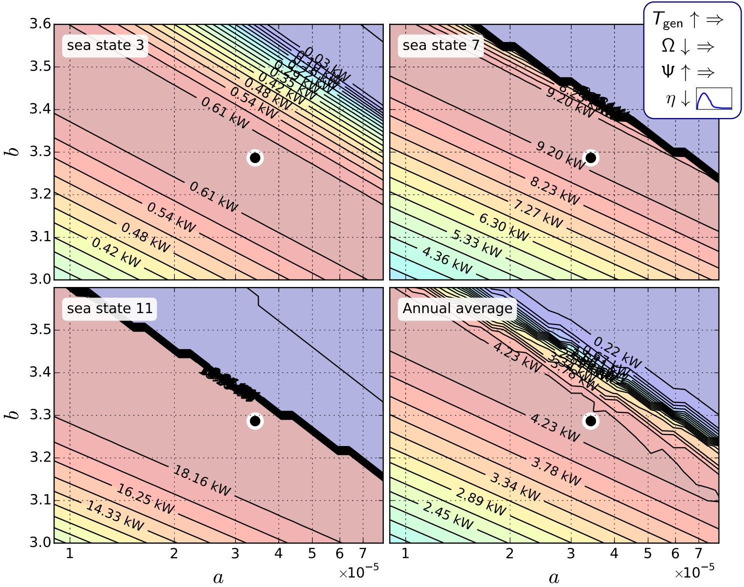
Generator feedback control law



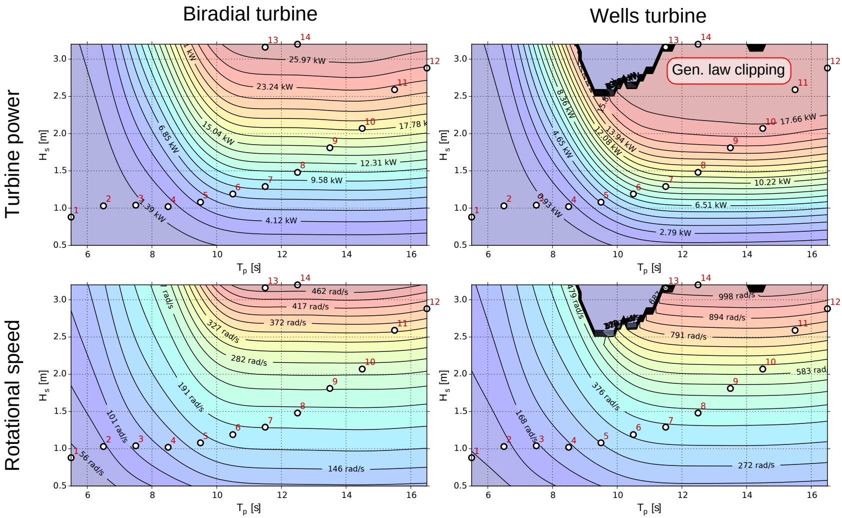
Biradial turbine power output sensitivity to "a" and "b"

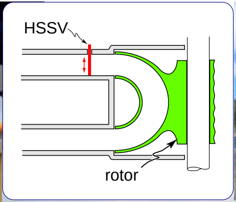


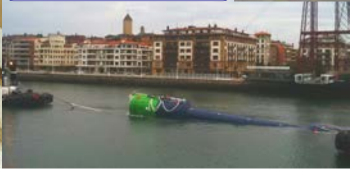
Wells turbine power output sensitivity to “a” and “b”



Comparison time-averaged turbine output power and rotational speed







- The HSSV valve of a biradial turbine with 0.5 m rotor diameter ▸ HSSV movie
- Latching with the HSSV will be tested at Mutriku, June-July 2017 (OPERA)
- HSSV will operate as safety valve in Oceantec MARMOK-A-5, October 2017 (OPERA). Buoy: 5 m diameter, 42 m in length and 80 tonnes weight.

Latching control and air compressibility

- **OWCs vs oscillating body WECs**
  - Latching on OWCs not so effective as in the case of an oscillating-body WECs
    - air compressibility has a spring effect
    - closing the HSSV does not stop the relative motion between the OWC and buoy
  - Decreases the forces resulting from latching in comparison with solid-body breaking
    - the valve surface area subject to the chamber pressure is a small fraction of the area of the OWC free surface
    - no impact forces - air compressibility spring effect
  - Removes the constraint of latching having to coincide with an instant of zero relative velocity between the floater and the OWC

Changes to the mathematical model of Mutriku

- Spar-buoy OWC - two body system **constrained to oscillate in heave**

$$\textbf{Buoy:}\quad (m_1 + A_{11}^{\infty})\,\ddot{x}_1 + A_{12}^{\infty}\,\ddot{x}_2 = -\rho_w g S_1\,x_1 - R_{11} - R_{12} + F_{d1} + p_{atm}\,S_2\,p^*$$

$$\textbf{OWC:}\quad A_{21}^{\infty}\,\ddot{x}_1 + (m_2 + A_{22}^{\infty})\,\ddot{x}_2 = \underbrace{-\rho_w g S_2\,x_2}_{\text{buoyancy}}\underbrace{-R_{21} - R_{22}}_{\text{radiation}}\underbrace{+F_{d2}}_{\text{diffraction}}\underbrace{-p_{atm}\,S_2\,p^*}_{\text{air chamber}}$$

- HSSV simulation requires a new definition of Ψ

$$\tilde{\Psi} = u\,\Psi = u\,\frac{p^*\,p_{at}}{\rho_{in}\,\Omega^2\,d^2}$$

where $u \in \{0,1\}$ is the **discrete** control (close/open)

- System of equations written as a 1st-order ODE

$$\dot{\mathbf{x}} = \mathbf{f}(t,\mathbf{x},u) \qquad (1)$$

$\mathbf{x}$ are called the states

- **Optimal control**
  - Maximization of a performance index based on the control u of the HSSV

$$\max\left(\int_0^T \mathcal{L}\left(\mathbf{x},\mathbf{u}\right)\mathrm{d}t\right)$$

    - Example: $\mathcal{L}\left(\mathbf{x},\mathbf{u}\right) = \dfrac{P_{\text{turb}}(t,\mathbf{x},u)}{\rho_{\text{atm}}\Omega_{\max}^3 d^5}$

  - Using the **Pontriagyn Maximum Principle** (PMP) the optimal problem is recast as

$$\max\left(\underbrace{\int_0^T \mathcal{L}\left(\mathbf{x}, u\right)\mathrm{d}t}_{\text{performance index}} - \underbrace{\int_0^{T_f} \boldsymbol{\lambda}^T\left(\dot{\mathbf{x}} - \mathbf{f}(t,\mathbf{x},u)\right)\mathrm{d}t}_{\text{constrained to system dynamics}}\right)$$

  - $\boldsymbol{\lambda}'$s are called adjoint variables

Solution of the Pontriagyn Maximum Principle

- The PMP shows that:
  - along the **optimal control path** $(\mathbf{x}, u)$ the Hamiltonian function $\mathcal{H}$

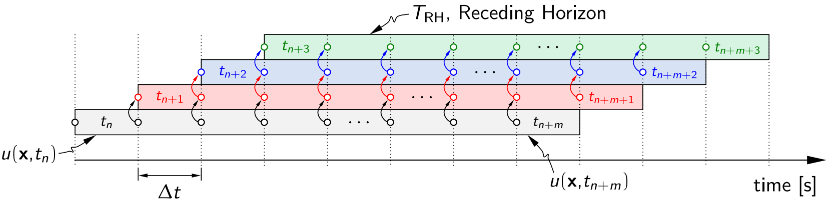
    $$\mathcal{H}\left(t, \mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}\right) = \boldsymbol{\lambda}^{\mathsf{T}}\, \mathbf{f}\left(t, \mathbf{x}, \mathbf{u}\right) + \mathcal{L}\left(\mathbf{x}, u\right),$$

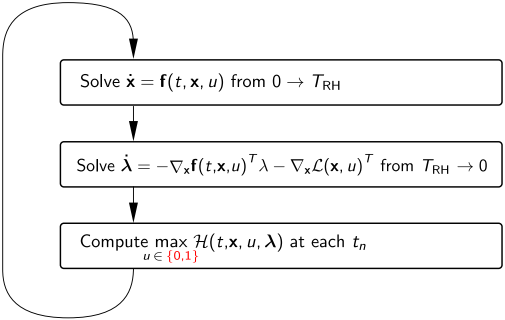
    is maximum for the optimal input u subjected to:

    - $$\left.\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(t, \mathbf{x}, u) \\ \mathbf{x}(0) &= \mathbf{x}_0 \end{aligned}\right\} \Rightarrow \text{forward solution}$$
    - $$\left.\begin{aligned} \dot{\boldsymbol{\lambda}} &= -\nabla_{\mathbf{x}} \mathbf{f}(t, \mathbf{x}, u)^{\mathsf{T}} \boldsymbol{\lambda} - \nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}, u)^{\mathsf{T}} \\ \boldsymbol{\lambda}(T_f) &= 0 \end{aligned}\right\} \Rightarrow \text{backward solution}$$

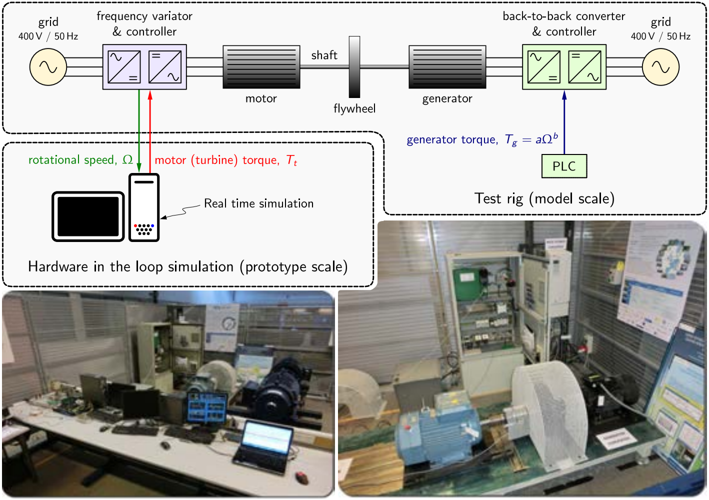
    for $t \in [0, T_f]$
  - **non-causal control** - $F_d(t)$ required to compute $\mathbf{f}$ for $t \in [0, T_f]$
- For complete details see the two classical books of Luenberger [5] and Bryson & Ho [6]

Receding Horizon control in a **real-time** framework



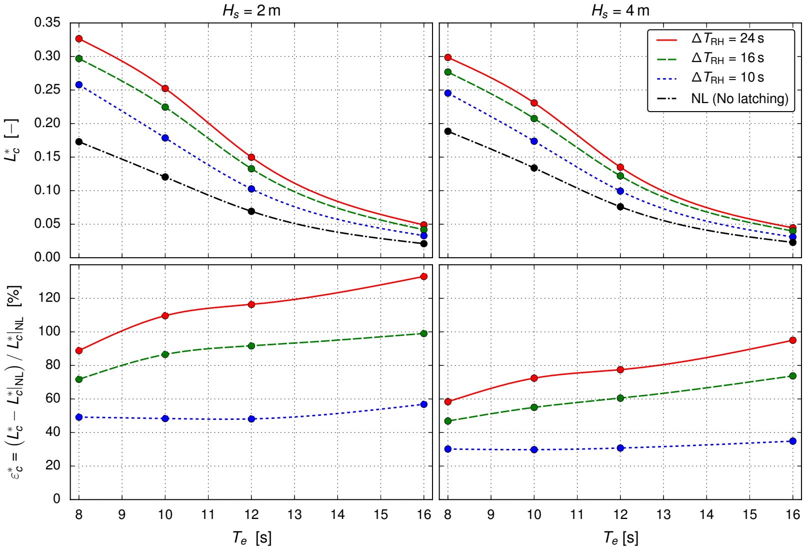


Hardware-in-the-loop tests at Tecnalia test rig

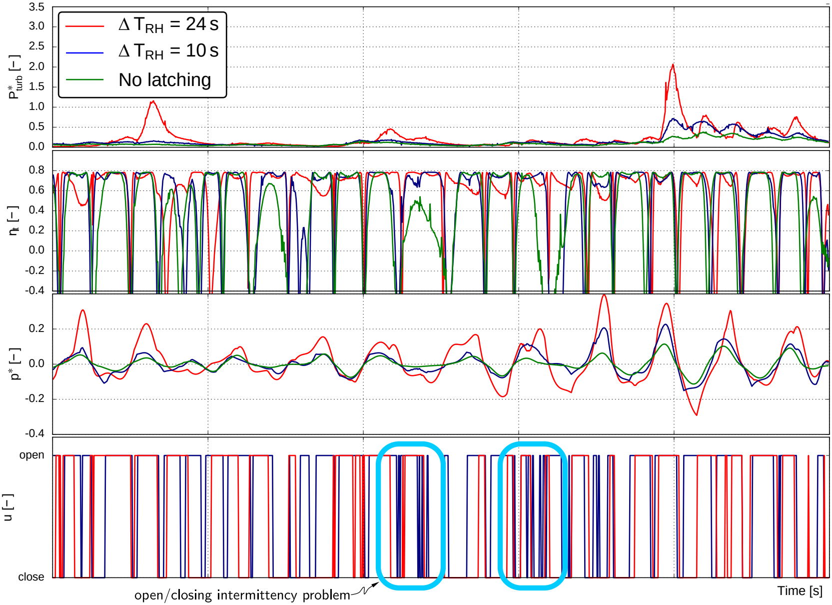


See [3, 4] for further details

Hardware-in-the-loop tests at Tecnalia test rig



Hardware-in-the-loop tests at Tecnalia test rig

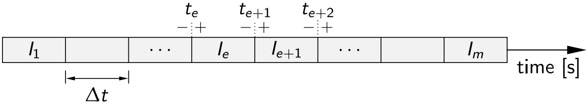


Lessons learned from the Tecnalia hardware-in-the-loop tests

- To improve stability, all state variables must have the same order of magnitude
- The algorithm sometimes opens/closes the HSSV intermittently during short periods (less than 1.5 seconds)
- The discrete nature of the numerical solution of the PMP problem implies a 2nd-order accuracy
- **A new challenge in the HSSV control**
  - The **sub-optimal problem** of open/closing the HSSV **only** at the end of each (**non-infinitesimal**) time step, $\Delta t = 0.1 \sim 0.2\, s$
  - No solution found in the searched bibliography!
    - We start the quest for a new solution method...

Discontinuous Galerkin Method

- The computational domain is discretized in small time-elements



where

  - State, adjoint and control variables approximated by a set of Legendre polynomials, $p_j(t)$ such that $x = \sum_j p_j(t)\tilde{x}_j$ in each time element
  - **The numerical solution is continuous within the time elements and allowed to be discontinuous across element boundaries**

- Solution of the **sub-optimal control problem** - compute $\mathbf{u}$ that maximizes the Hamiltonian $\mathcal{H}$ in a integral sense

$$\max_{\mathbf{u} \in \{0,1\}} \int_{t_e}^{t_{e+1}} \mathcal{H}(t, \mathbf{x}, \mathbf{u}, \boldsymbol{\lambda})\, \mathrm{d}t$$

Discontinuous Galerkin Method

- In the DG Finite Element Method, the original problem

$$\dot{\mathbf{x}} - \mathbf{f}(t, \mathbf{x}, \mathbf{u}) = 0$$

is replaced by a weak formulation

$$\int_{I_e} v_h \underbrace{\left(\frac{\mathrm{d}x_h}{\mathrm{d}t} - f\right)}_{\text{state equations}} \mathrm{d}t + \underbrace{v_h(t_e)\ [x_h(t_e^+) - x(t_e^-)]}_{\text{weakly enforced BC}} = 0$$

v_h is the so-called test function in the FEM framework

- Applying an affine transformation from $t \in I_e$ to $\tau \in [-1,1]$

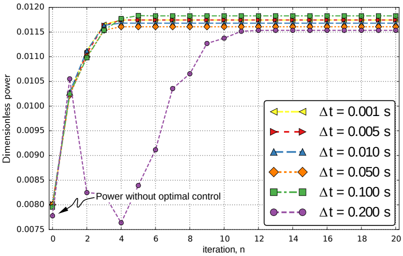
$$\int_{-1}^{1} \hat{v}_h \frac{\mathrm{d}\hat{x}_h}{\mathrm{d}\tau} \mathrm{d}\tau + \hat{v}_h(-1)\, \hat{x}_h(-1^+) = \frac{\Delta t}{2} \int_{-1}^{1} \hat{v}_h\, \hat{f}\, \mathrm{d}\tau + \hat{v}_h(-1)\, \hat{x}(-1^-) \qquad (2)$$

we write (2) as linear system $A\,\hat{x}_h = b(t,\mathbf{x},u)$ in each element

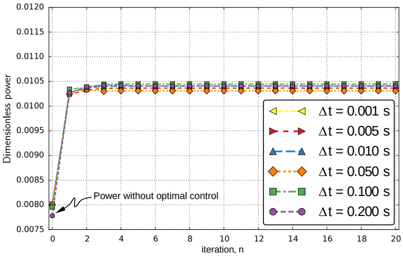
- To improve the convergence rate, a regularization term was introduced

$$\mathcal{L}(\mathbf{x}, u) = \frac{1}{T}\int_0^T \left(\frac{P_{\text{turb}}(t, \mathbf{x}, u)}{\rho_{\text{atm}}\Omega_{\text{max}}^3 d^5} + \varepsilon\,(1-u)^2\right) \mathrm{d}t$$

ε is a small constant

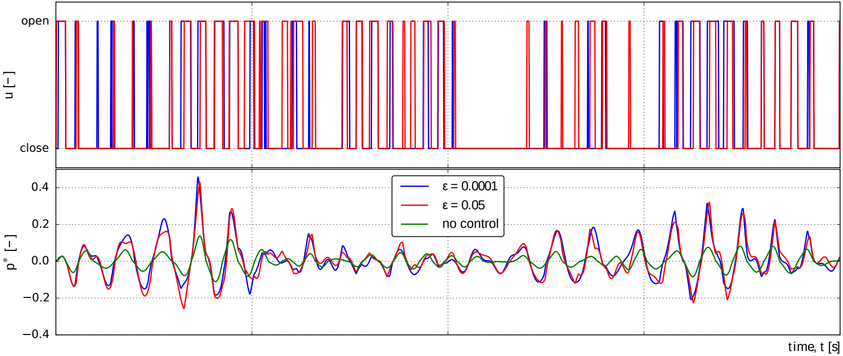


$\varepsilon = 0.0001$



$\varepsilon = 0.05$

Discontinuous Galerkin Method



Major conclusions

- Receding horizon optimal latching algorithm can be used to improve the OWC spar-buoy capture width
    - Receding horizon time interval between 10 to 24 s
- The Discontinuous-Galerkin solves the HSSV open/closing intermittency problem of the discrete optimal control
- **Probably the greatest advantage of the OWC technology**
    - Simple control of the available power to PTO system by using a HSSV
        - HSSV can be used for latching and control of the available power to the turbine and generator
- The proposed Discontinuous-Galerkin method is an efficient alternative to the well know Pseudo-Spectral Methods
    - High-order accuracy, mesh and polynomial (h-p) refinement, local nature of the solution, simple parallelization (real-time applications)

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